Low Wage Gig Sector Increase Wages in Indivisible Labor Monopsony Labor Market

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Abstract

Conventional economic theory tells us that introducing a competitor or a minimum wage can increase wages and employment in a labor market with a monopsony employer. In this paper, I explore the effect of a gig sector employer on wages in a formal sector with a single monopsony employer. Crucially, I assume that gig sector wages are lower than formal sector wages, but individuals can pick the number of hours worked in a gig sector but have to work either m hours or none in the formal sector. Let the formal sector wages under a monopsony market be w_f^*. Using a heterogenous agent model, assuming a log utility function, and applying numerical methods, I show that a gig sector paying $w_g = 0.75 * w_f^*$ will increase the formal sector wages by 37.3%. This new w_f' is 87.6% the wages in the formal sector under a two-firm oligopsony market. I have thus provided a theoretical argument that a gig sector paying low wages could improve welfare for those who do not participate in it.

1.Motivation and Intuition

In Funville, there is a Target, the sole employer[1]. As a monopsony power, it artificially pushes down wages. As a formal sector employer, it can hire workers for m hours or none. We assume that individuals have different tastes regarding consumption and leisure: only those who like consumption sufficiently would work. We assume no labor search costs, and anyone who is willing to work can. From standard economic theory, we know that the introduction of a Walmart or the institution of a minimum wage could increase wages and employment in the formal sector.

Suppose Lyft, a gig sector employer, starts operating in the area. One can spend as many hours working for Lyft as one wishes. We assume that Lyft's wages are lower than Target's wages. If Lyft were also a formal sector employer, nobody would work for them—if one could work m hours at Target but didn't, then why would one work m hours at Lyft for a lower wage? However, given that someone could work $m' < m$ hours at Lyft, unemployed individuals may choose to work for Lyft. In addition, those who really like consumption would want to work for Lyft on top of working for Target.

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[1] Another way to conceptualize this is to think of Target as the sole employer among low-skilled workers.

Neither of those would seem to directly impact wages and employment at Target. However, consider those people already employed at Target, who are now given this new option of gig sector work. Possibly, some would prefer to work fewer hours for a lower wage to enjoy more leisure. How would Target respond to this? Would wages in the formal sector increase? If so, by how much?

I explore this question using a theoretical model and numerical methods assuming a log linear utility function. Compared to the world with only one formal sector employer paying w_f^*, a gig sector paying 75% of that wage would increase the formal sector wage by 37.3%. This new formal sector wage is 87.6% the wage when there are two competing oligopsony firms. In other words, the existence of a gig sector is welfare improving in a monopsony labor market even for individuals who do not participate.

2.Conceptual Setup

We have a formal sector and a gig sector. In the formal sector, individuals are either employed and work m hours or are unemployed and work zero hours. The formal sector pays w_f using a production function $f_f(L) = AL_f^\gamma$, where L_f is the total hours of labor employed by the formal sector. When a gig sector exists, individuals devote any number of hours they want to gig work (which we will call g), subject to $\mathbb{1}_f m + g + l = T$, where l is leisure, $\mathbb{1}_f$ is an indicator function for participation in the formal industry, and T is normalized to one. The gig sector has production function $f_g(L_g) = w_g L_g$ with wage w_g. Throughout the model, we will ignore the demand side of the economy and assume that the price of the good produced remains constant regardless of quantity.

We have a continuum of individuals distributed on the open unit interval who differ in terms of their taste for consumption c and leisure l. They are indexed by their taste parameter α_i, where small α_i implies that they like consumption.

We first start with a world with only a formal sector employer. This means that, given a utility function, we can find the proportion of individuals that work in the formal sector (which I will denote by α without any subscripts) given some formal sector wage w_f. We do this by solving for the α_i level that makes someone indifferent between working and not. This yields a function that specifies formal sector employment rate $\alpha(w_f)$. The monopsony firm considers this and sets wages to maximize its profits. We will refer to the equilibrium employment rate without a gig sector as α^* and the wage w_f^*. I will use α' and w_f' to refer to the formal sector employment and wage when there is a gig sector.

Then, we introduce the gig sector with $w_g < w_f^*$. We are interested in the behavior of individuals who are already working in the formal sector. Namely, what proportion of those would enter the gig sector and drop out of the formal sector? We do that by comparing the utility of working only in the gig sector with the utility of working only in the formal sector[2]. The key idea is that the threat of these workers dropping out of the formal sector compels the monopsony employer to raise wages and retain employees.

We then plug $\alpha'(w_f, w_g)$, the new formal sector employment rate function, back into the

[2]We do this only for the subset of individuals who would not want to work in gig sector on top of working formal sector–these individuals really like consumption. The issue is that there might be a world where their optimal gig-sector only work schedule yields more utility that only working in the formal sector since they could work $m^* >> m$ hours and make more money than $m \cdot w_f$.

monopsony firm's profit maximization decision and observe its impact on wages and formal sector employment. We also solve for wages in a competitive labor market and an oligopsony market with two formal sector firms to benchmark the wage and employment effect of a gig sector employer.

3.Numerical Analysis with a Specific Utility Function

$$u_i(l,c) = (1-\alpha_i)\ln(k+c) + \alpha_i \ln(l)$$

3.1 Setup

One can think of k as the amount of money individuals have from non-wages sources or as some reservation utility. An individual works in the formal sector if $u_{if} = (1-\alpha_i)\ln(k + w_f m) + \alpha_i \ln(1-m) > (1-\alpha_i)\ln(k)$. Rearranging, this utility function yields the following $\alpha^*(w_f)$

$$\begin{aligned}
(1-\alpha)\ln(k+w_f m) &= (1-\alpha)\ln(k) - \alpha\ln(1-m) \\
\ln(k+w_f m) - \alpha\ln(k+w_f m) &= \ln(k) - \alpha\ln(k) - \alpha\ln(1-m) \\
\ln(k+w_f m) - \ln(k) &= \alpha[\ln(k+w_f m) - \ln(1-m) - \ln(k)]
\end{aligned}$$

$$\alpha(w_f) = \frac{\ln(k+w_f m) - \ln(k)}{\ln(k+w_f m) - \ln(1-m) - \ln(k)} \tag{1}$$

We then write out the firm's maximization problem:

$$\begin{aligned}
&\max_{w_f} A(m\alpha)^\gamma - w_f m\alpha = \\
&\quad Am^\gamma \left(\frac{\ln(k+w_f m) - \ln(k)}{\ln(k+w_f m) - \ln(1-m) - \ln(k)}\right)^\gamma - \frac{w_f m[\ln(k+w_f m) - \ln(k)]}{\ln(k+w_f m) - \ln(1-m) - \ln(k)}
\end{aligned} \tag{2}$$

There isn't a closed-form solution for w_f. We will solve it numerically in Matlab.

Gig sector

Individuals would work in the gig sector on top of the formal sector work if the optimal level of g in the following optimization is nonnegative:

$$\max_g \ (1-\alpha_i)\ln(k+w_f m + w_g g) + \alpha_i \ln(1-m-g)$$

Taking the FOC and rearranging

$$\begin{aligned}
\frac{w_g(1-\alpha_i)}{k+w_f m + w_g g} &= \frac{\alpha_i}{1-m-g} \\
w_g(1-\alpha_i)(1-m) - g w_g(1-\alpha_i) &= \alpha_i(k+w_f m) + \alpha_i w_g g \\
\alpha_i w_g g + g w_g(1-\alpha_i) &= w_g(1-\alpha_i)(1-m) - \alpha_i(k+w_f m) \\
g &= \frac{w_g(1-\alpha_i)(1-m) - \alpha_i(k+w_f m)}{\alpha_i w_g + w_g(1-\alpha_i)}
\end{aligned}$$

$$g = (1-\alpha_i)(1-m) - \frac{\alpha_i}{w_g}(k + w_f m) \tag{3}$$

We can rewrite the expression in terms of α_i to get the proportion of people who would take on two jobs (i.e., $g > 0$):

$$\alpha_i(k + w_f m) < (1-\alpha_i)(1-m)w_g$$
$$\alpha_i[(1-m)w_g + (k + w_f m)] < (1-m)w_g$$
$$\alpha_i < \frac{(1-m)w_g}{(1-m)w_g + (k + w_f m)} \tag{4}$$

Note that these individuals will not drop out of the formal sector. Someone only drops out when they don't like formal sector work sufficiently such that they would rather decrease their consumption and increase their leisure. These individuals want to do the exact opposite.

The next step is to solve for someone's utility if they only worked in the gig sector:

$$\max_g \ (1-\alpha_i)\ln(k + w_g g) + \alpha_i \ln(1-g)$$

Taking the FOC yields:

$$\frac{w_g(1-\alpha_i)}{k + w_g g} = \frac{\alpha_i}{1-g}$$
$$w_g(1-\alpha_i) - w_g(1-\alpha_i)g = \alpha_i k + \alpha_i w_g g$$
$$g = \frac{w_g(1-\alpha_i) - a_i k}{w_g} = (1-\alpha_i) - \frac{\alpha_i k}{w_g}$$

If they work in the gig industry, they will get utility

$$\begin{aligned} u_{ig} &= (1-\alpha_i)\ln(k + w_g g) + \alpha_i \ln(1-g) \\ &= (1-\alpha_i)\ln(k + w_g(1-\alpha_i) - a_i k) + \alpha_i \ln\left(1 - \frac{w_g(1-\alpha_i) - a_i k}{w_g}\right) \end{aligned}$$

An individual will quit the formal industry and join the gig industry if $U_{ig} - U_{if} > 0$ or

$$(1-\alpha_i)\ln(k + w_g(1-\alpha_i) - a_i k) + \alpha_i \ln\left(1 - \frac{w_g(1-\alpha_i) - a_i k}{w_g}\right) - \\ [(1-\alpha_i)\ln(k + w_f m) + \alpha_i \ln(1-m)] > 0$$

Once again, we would need a numerical approximation in order to find the α_i for the above to hold with equality.

3.2 Numerical Analysis: Methodology

I choose parameter values $k = 1$, $m = 0.6$, and $A = 30$. This yields an equilibrium formal sector wage of $w_f^* = 5.135$ when the gig sector is not present and an employment level of $\alpha^* = 0.6055$, close to the current U.S. employment levels. I solved for w_f^* and α^*

by maximizing the profit function by finding where the derivative is zero numerically, then verifying that the second derivative at the point is negative.

I examine scenarios where the gig sector wage is 50-90% of the pre-gig formal sector wages, incrementing in 5% intervals. I first solve for the optimal number of hours any given individual would spend in the gig industry using equation 3. I substitute that into the utility function to get the utility individual i receives from the gig sector U_{ig}. I then iterate through 90-150% of the old w_f^*, incrementing in 1% intervals, and calculate U_{if} for every given individual at that new formal sector wage. For individuals with α_i value higher than the right side of equation 4 (i.e., they do no work in the gig sector on top of the formal sector) I compare U_{ig} and U_{if} to check if they are better off working only in the formal industry or only in the gig industry. This will give me the new level of equilibrium employment rate under the new formal sector wage.

Then, taking the set of datapoints (w_f, α), I use a fourth-order polynomial approximation for the new employment function when a gig sector exists $\alpha_g(w_f)$. Then, I numerically maximize the new profit equation:

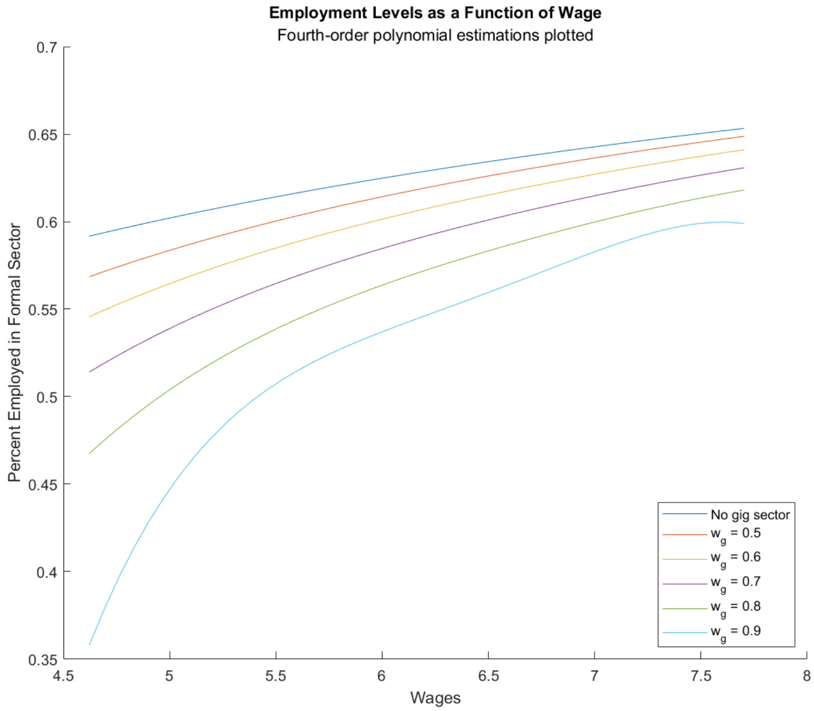
$$\max_{w_f} \pi_g = Am^\gamma \alpha_g^\gamma - w_f m \alpha_g \tag{5}$$

which will give me the new equilibrium employment level and wage given this set of parameters and the gig sector wage. Full Matlab code is available in the appendix.

3.3 Numerical Analysis: Results

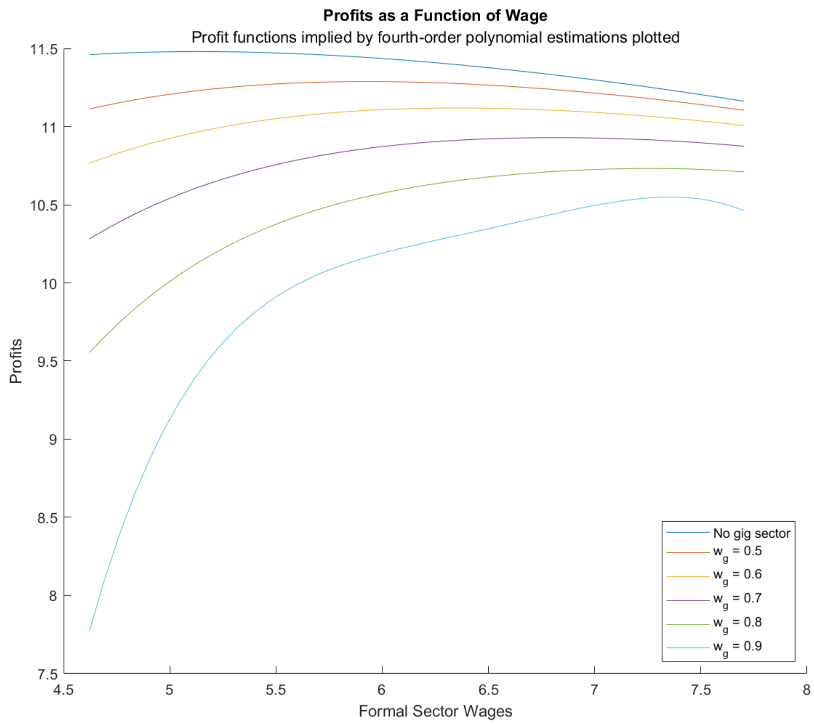
First, I verify that my fourth-order polynomials do an excellent job of estimating the employment function, reaching R^2 values larger than or equal to 0.9939 for all w_g values. Using the polynomial approximations, we see that the existence of a gig sector changes the population's formal sector employment rate given a wage.

Figure 1: Estimates of $\alpha(w_f, w_g)$



This changes the profit function and the profit-maximizing wage,

Figure 2: Profit functions at different gig sector wages



resulting in the following relationship between w_g and w_f':

As we can see, a flexible-hours gig sector with wages lower than the formal sector can still push up wages in a monopsony market. The effect on total employment in the formal industry is less clear.

The full simulation results are in the appendix in Table 8.

4. Benchmarking with an Oligopsony and Competitive Factor Market

We can see that a gig sector employer paying 75% the wages of the old formal sector wage could increase formal sector wages by 37.3%. However, how does that compare with the entrance of another firm or a perfectly competitive wage?

In competitive equilibrium, the firm makes zero profits. Using the same parameter values, we set equation 2 equal to zero and solve for w_f numerically to get 35.00. This would imply an employment rate of 77.13%. Thus, introducing a gig sector employer still leaves wages far from competitive.

We consider two firms j and k competing in an oligopsonistic factor market. We first rewrite equation 1 to be the wage as a function of total employment:

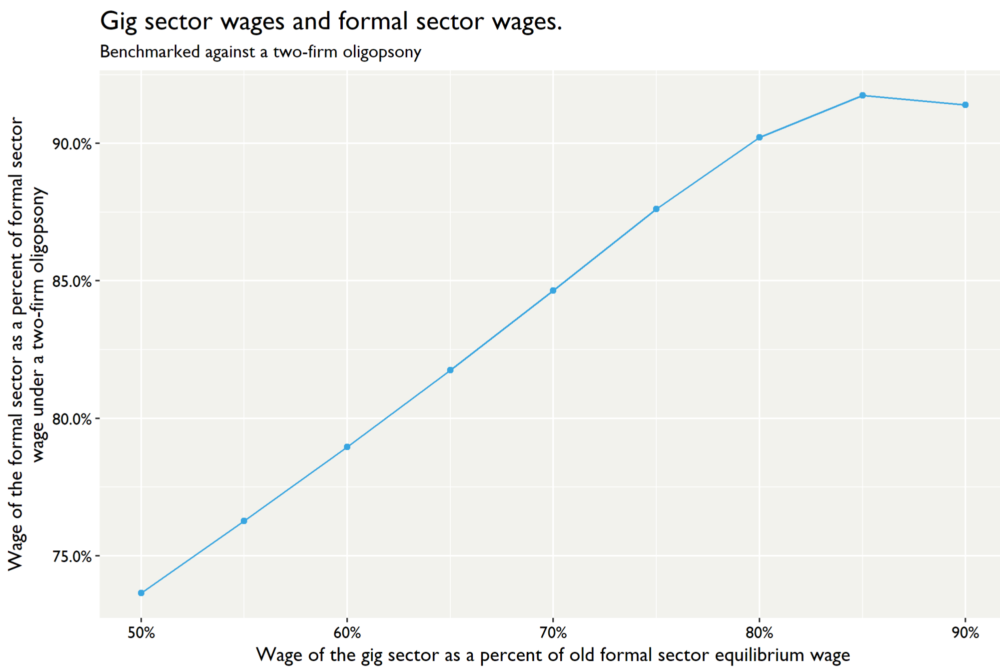
$$\begin{aligned}(1-\alpha)\ln(k+w_f m) &= (1-\alpha)\ln(k) - \alpha\ln(1-m)\\ (k+w_f m)^{1-\alpha} &= k^{1-\alpha}(1-m)^{-\alpha}\\ k+w_f m &> k(1-m)^{\frac{\alpha}{\alpha-1}}\\ w_f(\alpha) &= \left[k(1-m)^{\frac{\alpha}{\alpha-1}} - k\right] m^{-1}\end{aligned}$$

This yields a new profit maximization equation for firm j, who is now picking the amount of employees to hire (α_j) instead of the wage:

$$\max_{\alpha_j} \pi_j = Am^\gamma \alpha_j^\gamma - w_f(\alpha_j+\alpha_k)\alpha_j m \tag{6}$$

To solve a traditional Cournot equilibrium model, we need to find out the optimal response function for firm k as a function of how many individuals firm j hires, then set the wages and employment in the two firms equal to each other. We would need to use numerical methods again since there isn't a closed-form solution for the optimal response function. I use values for α_j from 0.25 to 0.4, incrementing by 0.0025, solve for the optimal α_k, then estimate the response function using a polynomial approximation.

Perhaps surprisingly, the response function appears to be completely linear ($\alpha_j = -0.8138\alpha_k + 0.5972$, $R^2 = 1.0000$). This yields $\alpha_j = 0.329$[3]. With total employment $\alpha = 0.658$, we have two-firm oligopsony formal sector $w_f^o = 8.049$. With this new measure, we can plot the effect of new formal sector wages as a percent of wages under an oligopsony market:



5.Empirical Methodology

Need sectors that is 1. have lots of market power with employers 2. exposed to gig work such that its workers might work at gig industries. This factors in education and family income.

6.Literature Review

This model builds on several established components. I build on indivisible labor literature started by Hansen (1985) and Rogerson (1988). More recent work on labor market models has incorporated both intensive and extensive margin shifts in the labor supply of

[3]Given this surprising result, I double-checked my calculation by assuming that the response function is a contraction (at least locally) and applied the Banach fixed point theorem. Starting with the previous level of $\alpha_j = 0.329$, I set $\alpha_k = \alpha_j$, find the optimal α_j given the α_k numerically, then repeat to find the fixed point. After five iterations, $\alpha_j \approx 0.3291595$ and $\alpha_j - \alpha_k \approx 0.00009339$.

heterogenous agents (Chang et al., 2018). However, models that split the economy into two different sectors based on whether or not labor is divisible appear to be much more rare. Vasilev (2017) explore such a two-sector model but adds the constraint that individuals working in the indivisible labor sector do not work in the divisible labor sector.

The majority of these models attempt to capture the behavior of the aggregate economy and business cycles. My model, however, is focused on the effect of a gig sector on a static monopsony labor market and uses numerical methods to understand how that impact compares to other market structures. Thus, it is also related to the literature on monopsony and oligopsony power in labor markets (Manning, 2021; Ashenfelter, Farber and Ransom, 2010; Chang and Tremblay, 1991).

Finally, this paper joins the growing literature examining the effect of the growing gig industry on the American labor market (Pew Research Center, 2021; Hall and Krueger, 2018; Abraham et al., 2018). In my model, the direct welfare benefits from the gig industry to individuals come from varying tastes over the total number of hours worked and the inability to choose that in a formal sector. Chen et al. (2019) takes a different approach and looks at the time-varying reservation wage. They estimate the welfare implications using Uber data and find that the flexible arrangement provided by gig work allows drivers to earn twice the surplus compared to a less flexible counterfactual. Other theoretical work has also explored the gig economy as an innovation in market design (Einav, Farronato and Levin, 2016).

7.Conclusion

Conventional economic theory tells us that the entry of a second firm and a minimum wage can push up wages in a monopsony labor market. Using a heterogenous agents model, I show that a gig sector could push up wages in a formal monopsony sector even if wages in the gig sector is lower. In fact, numerical simulations show that a gig sector employer paying 75% the wages of the old formal sector wage could increase formal sector wages by 37.3%. I provide a theoretical argument that the flexibility on total hours worked offered by the gig sector increases welfare of workers facing a monopsony employer.

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8.Appendix

Table 1: Full numerical results

w_g/w_f^*	w_g	w_f'	α'	R^2 of polynomial approximation	w_f'/w_f^*	α'/α^*
0.5	2.568	5.928	0.612	1.0000	1.154	1.011
0.55	2.825	6.139	0.612	1.0000	1.195	1.011
0.6	3.081	6.356	0.611	1.0000	1.238	1.010
0.65	3.338	6.580	0.611	1.0000	1.281	1.009
0.7	3.595	6.814	0.610	1.0000	1.327	1.007
0.75	3.852	7.052	0.609	1.0000	1.373	1.006
0.8	4.108	7.261	0.607	1.0000	1.414	1.003
0.85	4.365	7.385	0.603	0.9999	1.438	0.996
0.9	4.622	7.356	0.596	0.9939	1.432	0.985

```
Numerical Analysis
Tim Hua 2022
syms a_i w_f
assume(a_i, 'real')
assume(w_f, 'real')
k = 1
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```
m = 0.6
a_lnln = (log(k + w_f* m )- log(k))/(log(k + w_f*m) - log(1 - m) - log(k))
A = 30
gamma = 0.8
error = 0
pi = A*m^gamma*a_lnln^gamma - m*a_lnln*w_f
%Check for competitive equilibrium wage and employment
vpasolve(pi == 0, [1,50])
subs(a_lnln, w_f, vpasolve(pi == 0, [1,50]))

%Get one-sector monopsony wages.
piprime = diff(pi,w_f)
wstar = vpasolve(piprime == 0,w_f)
%Check 2nd order conditions:
sorder = vpa(subs(diff(piprime,w_f),w_f,wstar),2)
if sorder > 0
    disp(1)
    return
end
astar = subs(a_lnln,w_f,wstar)

wgmulti = [0.5:0.05:0.9]
drop = 0
for j = [1:length(wgmulti)]
    w_g = wgmulti(j)*wstar
    ghours = (1 - a_i) - a_i*k/w_g
    U_g_lnln = (1 - a_i)*log(k + w_g*ghours)+a_i*log(1 - ghours)
    %check if anyone would drop out under the old wstar
    if (1 - astar)*log(k + wstar*m) + astar*log(1-m) > ...
        (1 - astar)*log(k + w_g*subs(ghours,a_i,astar))+astar*log(1 - subs(ghours,a_i,as
        outputs(j,:) = [wgmulti(j), w_g, wstar, astar, -1]
        drop = drop + 1
        continue
    end

    multiplier = [0.9:0.01:1.5]
    for i = [1:length(multiplier)]
        wftemp = wstar*multiplier(i)
        tryhards = (1 - m)*w_g/((1 - m)*w_g + k + wftemp*m)
    %We search between tryhards and astar.
        U_f_lnln = (1 - a_i)*log(k + wftemp*m) + a_i*log(1-m)
        gigvuf = U_g_lnln - U_f_lnln
        %fplot(gigvuf,[tryhards astar])
        %Check this
        prop_emp_check = vpasolve(gigvuf == 0, a_i,[tryhards 1])
```

```
        if isempty(prop_emp_check) == 0
            prop_emp(i) = prop_emp_check
        else
            %Error
            disp(1)
            return
        end
    end
    wfvalues = double(wstar*multiplier)
    prop_emp = double(prop_emp)
    p = polyfit(wfvalues,prop_emp,4)
    a_polyfit_lnln = poly2sym(p, w_f)
    %Calculate R^2
    yfit = polyval(p,wfvalues)
    yresid = prop_emp-yfit
    SStotal = (length(prop_emp)-1)*var(prop_emp)
    SSresid = sum(yresid.^2)
    rsq = 1 - SSresid/SStotal
    %Calculating new optimal wage.
    pi_new = A*m^gamma*a_polyfit_lnln^gamma - m*a_polyfit_lnln*w_f
    piprime_new = diff(pi_new,w_f)
    wstar_new = vpasolve(piprime_new == 0,w_f,[wfvalues(1) wfvalues(length(multiplier))]
    %Check 2nd order conditions:
    sorder = vpa(subs(diff(piprime_new,w_f),w_f,wstar),2)
    if sorder > 0
        error = 1
        disp(1)
        return
    end
    astar_new = subs(a_polyfit_lnln,w_f,wstar_new)
    outputs(j,:) = [wgmulti(j), w_g, wstar_new, astar_new, rsq, wstar_new/wstar, ...
        astar_new/astar]
    profitsfuncs(j) = [pi_new]
    wagefuncs(j) = [a_polyfit_lnln]
end
%This takes a while to run, and I would clear the livescript output after
%it finishes running.

%Employment levels plot
figure
hold on
%plot(double(wstar*multiplier),prop_emp, LineStyle = "--")
xmax = double(wstar*multiplier(length(multiplier)))
xmin = double(wstar*multiplier(1))
fplot(a_lnln, [xmin xmax])
```

```
fplot(wagefuncs(1), [xmin xmax])
fplot(wagefuncs(3), [xmin xmax])
fplot(wagefuncs(5), [xmin xmax])
fplot(wagefuncs(7), [xmin xmax])
fplot(wagefuncs(9), [xmin xmax])
title("Employment Levels as a Function of Wage", ...
    "Fourth-order polynomial estimations plotted")
xlabel("Wages")
ylabel("Percent Employed in Formal Sector")
legend('No gig sector', 'w_g = 0.5', 'w_g = 0.6', 'w_g = 0.7', 'w_g = 0.8', 'w_g = 0.9',
    'Location','southeast')
hold off

%Profit functions plot
figure
hold on
xmax = double(wstar*multiplier(length(multiplier)))
xmin = double(wstar*multiplier(1))
fplot(pi, [xmin xmax])
fplot(profitsfuncs(1), [xmin xmax])
fplot(profitsfuncs(3), [xmin xmax])
fplot(profitsfuncs(5), [xmin xmax])
fplot(profitsfuncs(7), [xmin xmax])
fplot(profitsfuncs(9), [xmin xmax])
title("Profits as a Function of Wage", ...
    "Profit functions implied by fourth-order polynomial estimations plotted")
xlabel("Formal Sector Wages")
ylabel("Profits")
legend('No gig sector', 'w_g = 0.5', 'w_g = 0.6', 'w_g = 0.7', 'w_g = 0.8', 'w_g = 0.9',
    'Location','southeast')
hold off

%%%%%%%%%%%%%%%%%%%%
%Cournot Oligopoly
syms a_j
assume(a_j, 'real')
a_k_choices = [0.25:0.0025:0.4]
for i = [1:length(a_k_choices)]
    pi_oli = A*m^gamma*a_j^gamma - m*a_j*(k*(1 - m)^((a_j + a_k_choices(i))/ ...
        (a_j + a_k_choices(i) - 1)) - k)/m
    pi_oli_foc = diff(pi_oli, a_j)
    a_jstar_vec(i) = vpasolve(pi_oli_foc == 0, a_j,[0.05 0.5])
end
a_jstar_vec = double(a_jstar_vec)
```

```
p = polyfit(a_k_choices,a_jstar_vec,1)
yfit = polyval(p,a_k_choices) %Calculate intersection with y = x to get x = 0.329
yresid = a_jstar_vec-yfit
SStotal = (length(a_jstar_vec)-1)*var(a_jstar_vec)
SSresid = sum(yresid.^2)
rsq = 1 - SSresid/SStotal

%Verification using Banach fixed point theorem:
a_k = 0.329
pi_oli = A*m^gamma*a_j^gamma - m*a_j*(k*(1 - m)^((a_j + a_k)/(a_j + a_k - 1)) - k)/m
pi_oli_foc = diff(pi_oli, a_j)
a_jstar = vpasolve(pi_oli_foc == 0, a_j,[0.2 0.4])

a_k = a_jstar
pi_oli = A*m^gamma*a_j^gamma - m*a_j*(k*(1 - m)^((a_j + a_k)/(a_j + a_k - 1)) - k)/m
pi_oli_foc = diff(pi_oli, a_j)
a_jstar = vpasolve(pi_oli_foc == 0, a_j,[0.2 0.4])

a_k = a_jstar
pi_oli = A*m^gamma*a_j^gamma - m*a_j*(k*(1 - m)^((a_j + a_k)/(a_j + a_k - 1)) - k)/m
pi_oli_foc = diff(pi_oli, a_j)
a_jstar = vpasolve(pi_oli_foc == 0, a_j,[0.2 0.4])

a_k = a_jstar
pi_oli = A*m^gamma*a_j^gamma - m*a_j*(k*(1 - m)^((a_j + a_k)/(a_j + a_k - 1)) - k)/m
pi_oli_foc = diff(pi_oli, a_j)
a_jstar = vpasolve(pi_oli_foc == 0, a_j,[0.2 0.4])

a_k = a_jstar
pi_oli = A*m^gamma*a_j^gamma - m*a_j*(k*(1 - m)^((a_j + a_k)/(a_j + a_k - 1)) - k)/m
pi_oli_foc = diff(pi_oli, a_j)
a_jstar = vpasolve(pi_oli_foc == 0, a_j,[0.2 0.4])
a_jstar - a_k


%Extension to Stackleberg competition
%Not a real extension. I just did this and realized that I was doing
%Stackleberg instead of Cournot so I left it in here.
pi_oli = A*m^gamma*a_j^gamma - ...
    m*a_j*(k*(1 - m)^((a_j + a_k_polyfit)/(a_j + a_k_polyfit - 1)) - k)/m
pi_oli_foc_poly = diff(pi_oli, a_j)
a_jstar_polyfit_stackleberg = vpasolve(pi_oli_foc_poly == 0, a_j,[0.2 0.8])
```