Axioms and Theorems in Voting Theory with a Brief Biography of Kenneth May

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This paper provides a brief overview of axioms and their role in voting theory. Axioms are mathematically rigorous definitions of certain properties a voting scheme or social choice function. For example, neutrality describes what it means to treat all candidates equally. Axioms allow us to describe and compare different voting schemes, and we introduce theorems that highlight tradeoffs between axioms. Finally, we describe May's theorem in detail and give a brief overview of Kenneth May's life.

I. Introduction: What is voting theory, and what are its axioms?

Voting theory is concerned with *social choice*. While choice theory investigates how an individual makes choices between alternatives, voting theory focuses on how we can translate individual preferences over alternatives to a group preference. The most common manifestation of voting theory in our lives is, of course, elections. Voting rules are tools through which a society makes choices. As a result, voting theory is often referred to as social choice theory.

In the United States alone, there exists a large variety of different voting schemes. For example, New York City uses instant runoff voting to elect its mayor; Washington State uses a top-two primary for state-wise offices, and the country uses an extremely complicated system to elect its president. There are even more

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voting rules around the world such as Single transferable/non-transferable vote; first-past-the-post, and party-list proportional representation. This gives rise to the question: how do we think about and compare different voting schemes?

One such solution is to use axioms. Axioms are mathematically well-defined properties or criteria of voting rules. For example, I will give a rigorous definition of what it means to treat all voters equally. There are also axioms that describe who should be elected, whether or not a voter could lie about their preferences and get a better outcome, and the effect of adding more voters. We can classify a voting rule based on which axioms it satisfies. Since axioms are restrictive, sometimes we would find that there is only one voting rule which satisfies a set of axioms. Axioms are also normative in nature. There is no formula to tell you whether or not a voting rule should treat all voters equally, and it requires judgment on the part of the voter. Finally, an axiomatic approach to voting schemes is mathematical in its nature, yet it is not something that comes to mind when we consider what counts as "math." This discussion of axiomatic voting theory not only helps us better understand voting rules but also highlights how abstract mathematical thinking can be used as a tool to investigate very applicable real-world problems.

II. Notations and Definitions

The following notations, definitions, and statements of axioms follow largely from Zwicker and Moulin (2016) and help us establish the language with which we would learn about voting rules. To set up our voting model, we first introduce the static components:

- N is a finite set of n voters.
- A is a finite set of m alternatives (e.g. candidates in an election).
- Each voter i casts a ballot that lists their complete and transitive preferences $\succeq_i$ over A. We denote the $m!$ possible orderings by $\mathcal{L}(A)$.

- A voting profile $P = (\succeq_1, \succeq_2, \ldots \succeq_n)$ is the set of all ballots by all voters. We denote the set of all possible ballots by $\mathcal{L}(A)^n$.

We now introduce two definitions for what a voting rule can be:

Definition 1 *A social choice function, voting rule, or SCF is a function* $f : \mathcal{L}(A)^n \to 2^A$ *that returns some subset of* A*, the alternatives. If* $|f(P)| = 1$*, we say that* f *is single-valued or resolute and write* $f(P) = a$*.*

One can interpret the set $f(P)$ to be the winner of an election.

Definition 2 *A social welfare function or SWF* $g : \mathcal{L}(A)^n \to \mathcal{R}(A)$ *returns a ranking over the alternatives.*

In this paper, we would focus on single-valued social choice functions and not social welfare functions for the sake of brevity. We would also mostly ignore ties. We presume that we know each voter's complete preferences over A, even though many voting rules may not need that much information.

III. Axioms in Voting Theory

Now, armed with our new notational system, let's introduce several axioms. I would start with what are perhaps the more subjectively "obvious" axioms, and increase slowly in complexity.

The first set of axioms relates to *unanimity*. The basic idea is that we should select the alternative with the most support and not select an alternative that is clearly inferior to another alternative. This helps us eliminate perverse social choice functions such as "pick the alternative with an odd number of votes" or "pick the alternative with the third-highest vote count in a plurality election".

Axiom 1 *An SCF* f *is* unanimous *if, whenever there exists an alternative* a *such that for all voters* i *and all alternatives* $b \in A \setminus \{a\}$*,* $a \succ_i b$*, then* $f(P) = a$*.*

In other words, if a is the top choice for every voter, then the social choice function selects it.

Before introducing the next axiom, we first define what we mean by Pareto dominated: An alternative x is Pareto dominated if there exists some y such that $y \succ_i x$ for all voters i.

Axiom 2 *An SCF f is* Pareto *if $f(P)$ never contains a Pareto dominated alternative.*

The second set of axioms concerns monotonicity. While unanimity is concerned with absolute support, monotonicity looks at what happens when support for some alternative changes. Zwicker and Moulin (2016) lists more than five different definitions of monotonicity, and I will mention two here.

Axiom 3 *An SCF f satisfies* participation *if whenever a profile P is modified to P' by adding a ballot $\succeq_i$, then $f(P') \succeq_i f(P)$.*

In other words, adding voters that have a certain set of preferences can only weakly shift the social choice towards that set of preferences.

Axiom 4 *An SCF f satisfies* strong or Maskin monotonicity *if whenever a profile P is modified to P' where, for some voter i and all alternatives j such that $f(P) \succeq_i j$ and the new preferences $\succeq_i'$ satisfies $f(P) \succeq_i j \Rightarrow f(P') \succeq_i' j$, then $f(P) = f(P')$.*

In plainer words, if some voter's preferences change but the winning candidate does not lose any support, then that candidate should still win.

The third pair of axioms relates to equality. The first states that all voters are equal:

Axiom 5 *An SCF f is* anonymous *if each voter plays the same role. That is, for a given P, if P' is obtained by swapping i and j's ballots (i.e. $\succeq_i' = \succeq_j$ and $\succeq_j' = \succeq_i$), then $f(P) = f(P')$. A voting rule is* dictatorial *if $f(P)$ is the top choice of some voter i.*

Lemma 1 *Voting in the U.S. presidential election is not anonymous.*

It's easy to see that, if a ballot in a swing state is swapped with a ballot in a non-swing state, it could change the outcome of the election. When social choice functions are anonymous, we don't need to know voter i's preferences. We only need to know how many voters hold a certain preference $\succeq_i$. Thus, we can denote all voting profiles P with profile tables such as the following:

TABLE 1—EXAMPLE OF A PROFILE TABLE WHERE $m = 4$ AND $n = 22$

Number of voters	10	8	3	1
First place	a	b	d	c
Second place	b	a	b	d
Third place	c	c	a	b
Fourth place	d	d	c	a

In some papers, anonymity is assumes, and a profile p is defined as if it is a profile table. That is, p is a vector in $\mathbb{Z}_+^{|A|!}$ (where $\mathbb{Z}_+$ denotes the set of nonnegative integers) that counts the number of voters that hold one of the $|A|!$ possible preference relations over the set of alternatives (Can, Csóka and Ergin, 2021).

After introducing the mathematical definition of treating all voters equally, we now introduce the corresponding definition for treating candidates equally:

Axiom 6 *An SCF f is* neutral *if alternatives are interchangeable. That is, if $f(P) = a_1$, and P' is obtained by swapping a_1 and a_j in every voters' preferences, then $f(P') = a_j$. f is imposed if there exists some $a \in A$ such that $f(P) \neq a$ for all $P \in \mathcal{L}(A)^n$.*

We now introduce the unrestricted domain quality, which is a much more restrictive axiom than it looks on the surface.

Axiom 7 *We say that an SCF f has the* unrestricted domain quality *if it is resolute for all $P \in \mathcal{L}(A)^n$.*

A significant portion of voting theory deals with situations where the unrestricted domain quality does not apply. Spacial models of voting is such an example. In spacial models of voting, candidates and voters have views on certain issues that places them in a (possibly multidimensional) metric space. Then, voters' preferences over voters is a function of their distance to the candidates. One could vary the distribution of voter's views (some examples are uniform distributions, multivariate normals, and bimodal distributions for polarized societies) as well as the metric to explore different scenarios. We refer to Merrill (2014) and Enelow and Hinich (1984) for further information.

Another way the unrestricted domain quality is restrictive is that we often want to elect more than one candidate at once (in parliamentary systems, for example). Nonetheless, resoluteness is still a key axiom in the social choice theory and lends itself to interesting analysis.

We now introduce another restrictive but mathematically fascinating axiom:

Axiom 8 *When a voter i changes their preferences from $\succeq_i$ to $\succeq_i'$, and the resulting profile changes from P to P' as a result, we say an SCF f is* single-voter strategy-proof *if $f(P) \succeq_i f(P')$.*

We think of $\succeq_i$ as voter i's sincere preferences. Single-voter strategy-proof means that voter i cannot cast a ballot $\succeq_i'$ that misrepresents their preferences, and then obtain some result that is strictly better off than what they would have had they submitted their sincere preferences (i.e., $f(P') \succ_i f(P)$).

IV. Campbell-Kelley and Gibbard-Satterthwaite Theorems

Now that we are familiar with the basic axioms and notation, we can start examining the relationship between them. In the beginning, I noted that axioms are restrictive and sometimes only one type of voting rule satisfies a certain axiom. We will see two quick examples here. I will omit the proof in both cases.

Theorem 1 *(Gibbard-Satterthwaite, or GST) Any resolute, nonimposed, and strategyproof SCF for three or more alternatives must be a dictatorship.*

The GST is less famous than Arrow's impossibility theorem, which is focused on SWFs instead of SCFs, but it is nonetheless illuminating on how restrictive strategyproofness is. The fact that individuals can manipulate the outcome of an election by misrepresenting their preferences has long been known to those who study voting behavior. In 1283, Roman Lull mentioned how voters ought to "take an oath to tell the truth" before voting. In 1434, Nicolaus Cusanus described a method of voting analogous to the Borda count, and stated that "no other outcome is possible, *if the electors act according to conscience*, than the choice of that candidate adjudged best by the collective judgment of all present." (Empasis added) (Barberà, 2011; McLean and Urken, 1995, p.78).

An interesting fact I discovered while reading older texts on social choice is how they are concerned with electing the *right* person. For example, Condorcet (1785) argued that, because of the law of large numbers, if each voter has a less than $\frac{1}{2}$ chance of making the correct decision, then the whole group is likely to make the incorrect decision. It follows that "it could be dangerous to give a democratic constitution to a stupid group of people" (p. 25) [1]. Whereas modern discussion of social choice is more focused on electing a person that "best represents the group's preferences." This shift away from normative language is a part of a larger trend in the social sciences and it is exciting that it could also be found in discussion of voting theory.

A proof of GST is available in Zwicker and Moulin (2016).

Theorem 2 *(Campbell-Kelley) If there exists a Condorcet winner—some candidate that beats every other candidate in all pairwise contests (i.e., $x : f(\{x, y\}) = x$ for all $y \in A \setminus \{x\}$ where f is simple majority rule)—then picking that winner is the unique resolute, anonymous, neutral, and strategyproof SCF (given three or more candidates and an odd number of voters)*

Here, we restricted our domain in order to fulfill all of the other axioms. Proof is

[1] Original french text: "*on voit qu'il peut être dangereux de donner un constitution démocratique à un peuple sans lumières*"

available at Campbell and Kelly (2003). A comprehensive overview of which axioms a voting rule complies with can be found on the wikipedia page for the Borda Count, and I've attached a screenshot of it in Figure 1 Wikipedia contributors (2021*a*):

System	Mono-tonic	Condorcet winner	Majo-rity	Condorcet loser	Majority loser	Mutual majority	Smith	ISDA	LIIA	Independence of clones	Reversal symmetry	Participation, consistency	Later-no-harm	Later-no-help	Polynomial time	Resol-vability
Schulze	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No	No	No	Yes	Yes
Ranked pairs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	Yes	Yes
Split Cycle	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No	No	No	Yes	No
Tideman's Alternative	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	No	No	No	Yes	Yes
Kemeny–Young	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	No	No	No	Yes
Copeland	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	Yes	No	No	No	Yes	No
Nanson	No	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No	No	Yes	Yes
Black	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No	Yes	No	No	No	Yes	Yes
Instant-runoff voting	No	No	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No	Yes	Yes	Yes	Yes
Smith/IRV	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	No	No	No	Yes	Yes
Borda	Yes	No	No	Yes	Yes	No	No	No	No	No	Yes	Yes	No	Yes	Yes	Yes
Geller-IRV	No	No	Yes	Yes	Yes	Yes	No	No	No	No	No	No	No	No	Yes	Yes
Baldwin	No	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No	No	No	Yes	Yes
Bucklin	Yes	No	Yes	No	Yes	Yes	No	No	No	No	No	No	No	Yes	Yes	Yes
Plurality	Yes	No	Yes	No	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes
Contingent voting	No	No	Yes	Yes	Yes	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes
Coombs[3]	No	No	Yes	Yes	Yes	Yes	No	No	No	No	No	No	No	No	Yes	Yes
MiniMax	Yes	Yes	Yes	No	No	No	No	No	No	No	No	No	No	No	Yes	Yes
Anti-plurality[3]	Yes	No	No	No	Yes	No	No	No	No	No	No	Yes	No	No	Yes	Yes
Sri Lankan contingent voting	No	No	Yes	No	No	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes
Supplementary voting	No	No	Yes	No	No	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes
Dodgson[3]	No	Yes	Yes	No	No	No	No	No	No	No	No	No	No	No	No	Yes

FIGURE 1. A summary of voting rules and the axioms they fulfill

We note that, on the table, participation is the only criterion that appears on both the table and our discussion. However, almost all of these voting rules satisfy unanimity, anonymity, and neutrality. Some of the axioms listed are different versions of what was previously covered: Majority is simply a different type of unanimity axiom (If a candidate receives over 50% of the vote in plurality rule, that candidate must be elected) (Wikipedia contributors, 2021*b*); Resolvability is a probabilistic version of resoluteness (Wikipedia contributors, 2021*c*), and Wikipedia contributors had their own definition of monotonicity. The axioms I presented tend to be more fundamental and less applicable to comparisons of actual voting rules. However, I believe that they serve as a great introduction to axioms and the axiomatic framework precisely because of their simplicity.

V. May's Theorem

Here, we will show and prove one of the simpler theorems from the Golden age of social choice theory: May's theorem (1952).

Theorem 3 *In the two candidate case, a social welfare function is resolute, neutral, anonymous, and positively responsive if and only if it is simple majority rule.*

Before we proceed with the proof, a few points of clarification. I stated the theorem in terms of an SWF as opposed to an SCF because it could result in the group being indifferent between either alternative —something that May defined to also be a resolute outcome (May, 1952). Indifference is a form of order on the two alternatives, hence the SWF designation.

In the theorem, positive responsiveness is a form of monotonicity defined for the two-alternative case. The social welfare function f satisfies positive responsiveness in a two-alternative (a and b) election if, under some profile P where the alternative a is preferred to or is indifferent to b by the group, whenever a voter who did not strictly prefer a to b switches their preference to prefer a over b and we obtain to P', then under P' the group must prefer a over b. In mathematical notation, if $f(P) = \mathcal{R}(A) = a \succeq b$, and for some voter i such that $b \succeq a$, we change i's preferences such that $a \succ b$, then under the new profile P', $f(P') = a \succ b$.

Proof of May's Theorem: First, it's clear that simple majority rule is anonymous, neutral, resolute (when we consider a tie a resolute outcome), and positively responsive.

We see that anonymity implies that we can use profile tables to express everyone's preferences and that the SCF is dependent only on the total number of votes for either candidate. If there is some voting rule that satisfies the four axioms and is not simple majority, then there must be some case where a gets more votes than b, but b is preferred to a. Now we add votes to b until it has exactly as many votes as a does in the beginning. Positive responsiveness means that b would still be preferred to a, but neutrality would imply that a is preferred to b since we are in the exact same situation as before, but with the role of a and b swapped. Thus we reach a contradiction and that there must not be another voting rule that satisfies these criteria other than simple majority rule. ■

May's theorem might seem very obvious, but it does help us confirm that simple majority rule is supported by our axiomatic framework and that indirectly lends credibility to the framework itself. Moreover, it provides an argument for us to reduce multicandidate election down to pairwise contests since we know how to decide between two candidates.

VI. The Life of Kenneth May

The final section of the paper will be a historical note on Kenneth May's life. The information below is from Jones, Enros and Tropp (1984) unless otherwise stated.

JULY 8, 1915–DECEMBER 1, 1977

FIGURE 2. PHOTO SOURCE: (TROPP, 1979)

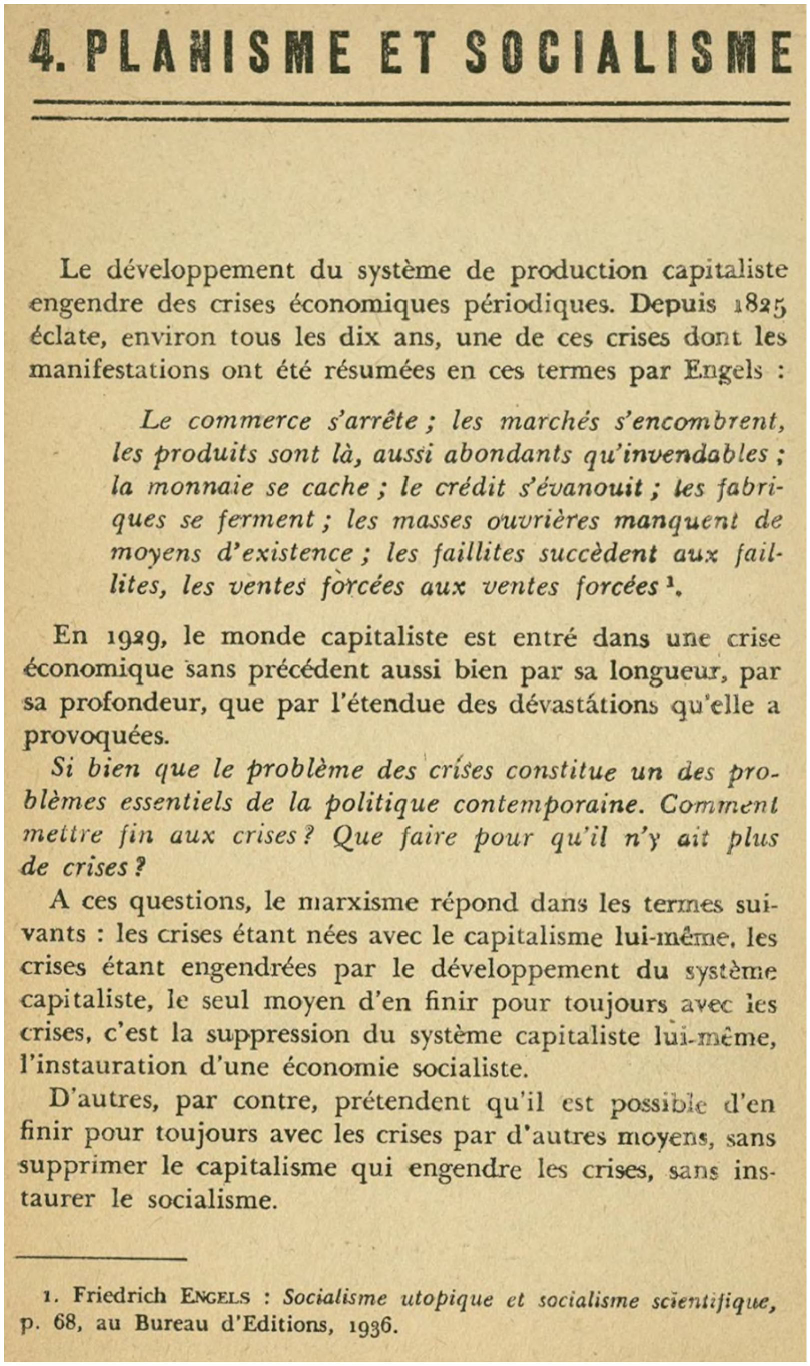
Kenneth May was born July 8th, 1915 in Oregon. He went to U.C. Berkeley for college, where he developed an interest in "the use of statistics as an element in national planning" (p.361). He joined the communist party in college and graduated in 1936 with a degree in mathematics.

After graduating, he took on a fellowship at the Institute of Current World Affairs (ICWA) to study science and technology in Soviet Russia. However, he found it difficult to obtain a visa to study in Russia, and instead first went as a tourist in 1937. He spent the rest of his time in Europe studying at London, taking Russian courses at the School of Slavonic Studies and courses in statistics at University College (now commonly known as UCL), as well as classes at the London School of Economics. May spent a lot of time studying statistical methods under R.A. Fisher, the inventor of Fisher's exact test, Fisher's principal, Fisher's inequality, F-distribution, etcetera. Fisher, however, was also the head of the Eugenics department and a massive racist. May also took courses on economic planning at the London School of Economics. At the time, F.A. Hayek taught at LSE, and May had thought to take one of his courses about the problems of a collectivist society (we are uncertain if he did). There was a vibrant intellectual debate around the merits of economic planning at the time in London.

In July of 1938, May married Ruth McGovney in London. His father so opposed the married that he traveled to London in a failed attempt to stop him. ICWA forbids its fellows from marrying and he lost his ICWA fellowship as a result and moved to Paris to study at Sorbonne University. At night, he also attended courses at the Worker's University (l'Université Ouvrière). To give readers an idea of what the atmosphere at the Worker's University is like, I found a pamphlet from the school titled "The Big Problems of Contemporary Politics: 4. Planned Economy and Socialism" (Figure 3 and 4)

A rough translation of the text in Figure 4 is in the appendix, but I will quickly summarize it here: Fajon first mentions recessions and their devastating impact on the economy, as well as the 1929's recession that is "without precedent in its

Figure 3. Photo source: (Fajon, 1938)



4. PLANISME ET SOCIALISME

Le développement du système de production capitaliste engendre des crises économiques périodiques. Depuis 1825 éclate, environ tous les dix ans, une de ces crises dont les manifestations ont été résumées en ces termes par Engels :

> *Le commerce s'arrête ; les marchés s'encombrent, les produits sont là, aussi abondants qu'invendables ; la monnaie se cache ; le crédit s'évanouit ; les fabriques se ferment ; les masses ouvrières manquent de moyens d'existence ; les faillites succèdent aux faillites, les ventes forcées aux ventes forcées*[1].

En 1929, le monde capitaliste est entré dans une crise économique sans précédent aussi bien par sa longueur, par sa profondeur, que par l'étendue des dévastations qu'elle a provoquées.

Si bien que le problème des crises constitue un des problèmes essentiels de la politique contemporaine. Comment mettre fin aux crises ? Que faire pour qu'il n'y ait plus de crises ?

A ces questions, le marxisme répond dans les termes suivants : les crises étant nées avec le capitalisme lui-même, les crises étant engendrées par le développement du système capitaliste, le seul moyen d'en finir pour toujours avec les crises, c'est la suppression du système capitaliste lui-même, l'instauration d'une économie socialiste.

D'autres, par contre, prétendent qu'il est possible d'en finir pour toujours avec les crises par d'autres moyens, sans supprimer le capitalisme qui engendre les crises, sans instaurer le socialisme.

1. Friedrich ENGELS : *Socialisme utopique et socialisme scientifique*, p. 68, au Bureau d'Editions, 1936.

FIGURE 4. PHOTO SOURCE: (FAJON, 1938)

length and depth." It then asks the rhetorical question: how do we stop these crises? The answer is that these crises are "born with capitalism itself; they are created by the growth of the capitalist system, and the only way to end these crises forever is the abolition of the capitalist system itself and the institution of a socialist economy." Doesn't this remind you of rhetoric surrounding abolishing or defunding the police in 2020 and 2021? If I had written something to the tone of: The reason innocent Black men in this country gets shot by police officers is that police system is inherently racist, "and the only way to end these [shootings] forever is the abolition of the [police] system and the institution of a [system of community-based care]." You may think that I am quoting from a *New York Times* opinion article like Kaba (2020).

A discussion of the U.S. police system is obviously outside the scope of this paper, but my point is that the rhetoric used by left-leaning activists today has a long history behind them, and it is not unlikely that Kenneth May used them almost ninety years ago.

After spending the winter in Paris, May traveled once more to the Soviet Union, before returning to U.C. Berkeley to finish his doctoral degree in mathematics. He once again joined the communist party and conducted some official business on its behalf, which was reported in the newspapers, which led to the events found in Figure 5.

In response to his father, May issued the following statement (Figure 6) in the San Francisco Chronicle saying that he believes "that the communist party is the greatest force for good and that only through socialism can the people of the United States solve the problem of unemployment, poverty, and war" (p.4) (May, 1940).

Communism is frequently associated with anti-Americanism, but the ideology at its core does not belong to any nation. It is simply a way of organizing an economy and a polity and there are people (maybe more back in 1940 than today) that truly believed that it is a better system than capitalism and that it would

The New York Times

FRIDAY, SEPTEMBER 27, 1940

Professor May Disowns Communist Son; 'Only Honorable Course,' Says Californian

Special to THE NEW YORK TIMES.

BERKELEY, Calif., Sept. 26—Samuel Chester May, veteran University of California professor and a leader in State affairs, "disowned and disinherited" his son, Kenneth, today because he is an avowed Communist.

The youth, who is a teaching assistant at the university, is campaign manager for the Communist party in Alameda County. He made his views public last night at a meeting of the Berkeley Board of Education, when he appeared to speak in behalf of a petition for permitting the use of Berkeley school buildings for Communist meetings.

"I have been dreading this break for weeks," said Professor May. "I have been expecting it for a long time. The break came privately some time ago, but now it must come publicly.

"Every one who knows me knows that my views are contrary to those of Kenneth. So decided are my views that I have completely disowned and disinherited Kenneth."

In addition to his post as head of the university's Bureau of Public Administration Professor May is vice chairman of the State Defense Council. He went to Sacramento today and after a conference with Governor Olson explained his attitude toward his son and asserted he had taken the only honorable course in repudiating him "for his espousal of the cause of communism."

"It is just one of those things that may happen to any father," he said. "Any one who has children can understand.

"For twenty years I've been fighting communism. I have students scattered all over this country who know how I stand on radicalism and communism, so I don't believe any one will question my position.

"When I became convinced my son had become an irreconcilable Communist I took the only honorable course consistent with my personal views and the position I hold as an executive of the defense council."

Kenneth May was graduated from the University of California in 1936 after a brilliant campus record. He was a Phi Beta Kappa student and a member of the Golden Bear Honor Society and the Student Judicial Council. In competition with students throughout the United States he won a scholarship under the Institute of World Affairs and studied in Europe for two and a half years, for five months in Russia.

Following his return he was made a teaching assistant in mathematics at the university. His wife, the former Miss Ruth McGovney, daughter of Professor Dudley O. McGovney of the university, is a teacher in Oakland High School.

After the board of education meeting he said:

"Some one had to take the action I did, and what happens cannot be helped."

FIGURE 5. SOURCE: (, 1940)

Statement Issued

Kenneth May Defends Membership in Party

Kenneth May, in a statement defending his membership in the Communist party, last night made no specific comment on the action of his father, Professor Samuel Chester May, in disowning him for his beliefs.

He took occasion to deny that his wife, who is the former Ruth McGovney, daughter of Professor Dudley O. McGovney of the university and a teacher in Oakland high schools, is a Communist.

May said:

"I first joined the Communist party as an undergraduate at the University of California because I found by actual experience in student activities that the Communists were consistent and uncompromising fighters for the interests of the students and against reaction within and without the university.

"I became convinced that the Communists were correct in their analysis of the university administration and the social system of which the university is a part.

"As a member of the Communist party I have seen how the party works everywhere to preserve and extend democracy, protect the living standards of the people, build the trade unions and keep this country out of war.

"During a stay of two years in Europe I observed the Communist parties in France and England in action and visited a number of other countries, including the Soviet Union. All these experiences convinced me that the Communist party is the greatest force for good in the world and that only through Socialism can the people of the United States solve the problem of unemployment, poverty, oppression and war.

"As a patriotic American I have no other choice but to work for what I consider to be the best interests of my country and the world."

FIGURE 6. SOURCE : (MAY, 1940)

bring about a prosperous society. May is one of them.

As a result of his communist activities, Berkeley dismissed him from his teaching assistantship and May left the school to work for the communist party full-time. Soon after, war broke out and he joined the 87th Mountain Infantry in the Army. He was first deployed at Kiska as a part of the Aleutian campaign. May eventually found himself in Italy and once more on the news. This time it was on page four of the *Stars and Stripes* Mediterranean (Rome) on March 3, 1945. The article detailed how he struggled to enlist because of his affiliation with communist parties, how he impressed a Lt. General with his wits, and his heroic actions in Italy where he rescued someone from a knocked-out bulldozer (Foisie, 1945).

May returned from the War and finished his Ph.D. in mathematics at U.C. Berkeley in 1946. He then went to Carleton College to work as an Assistant

Professor before leaving for the University of Toronto in 1966. Outside of his brief yet significant contributions to social choice theory, May was mostly known as a Historian of Mathematics. He founded the journal *Historia Mathematica*, which is still in circulation today (International Commission on the History of Mathematics, n.d.).

VII. Conclusion

This paper introduces social choice theory and gave several axioms that help us categorize and understand voting systems through mathematical reasoning. I also introduce three theorems that illustrate how axioms work in practice. Finally, I ended on a historical note and chronicled the life of Kenneth May: a patriot, communist, mathematician, and historian.

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Translation of Fajon 1938

Figure 4 is a scan of page three of Fajon (1938).

The development of the capitalist production system creates economic crises periodically. Since 1825, about every ten years, one of these crises breaks out whose's properties are summarized in these terms by Engels:

Trade stops; the markets are crowded, the products are there, as abundant as they are unsaleable; currency is nowhere to be found; credit vanishes; factories close; the working masses have no livelihood; bankruptcies follow bankruptcies, forced sales follow forced sales [2]

In 1929 the capitalist world entered a crisis unprecedented economic not only by its length and its depth, but also by the extent of the devastation it has provoked.

Since the problem of economic crises constitutes one of the essential problems of contemporary politics, how do we end these crises? What do we do so that there are no more of such crises?

[2]Friedricli Engels: Utopian socialism and scientific socialism, p. 68, at the Publishing Office, 1936

To these questions, Marxism answers in the following terms: the crises are born with capitalism itself; they are created by the growth of the capitalist system, and the only way to end these crises forever is the abolition of the capitalist system itself and the institution of a socialist economy.

Others, on the other hand, claim that it is possible to end these crises forever by other means, without suppressing the capitalism that breeds these crises and instituting socialism.